When a Near Collinearity Should and Should not be made Perfect

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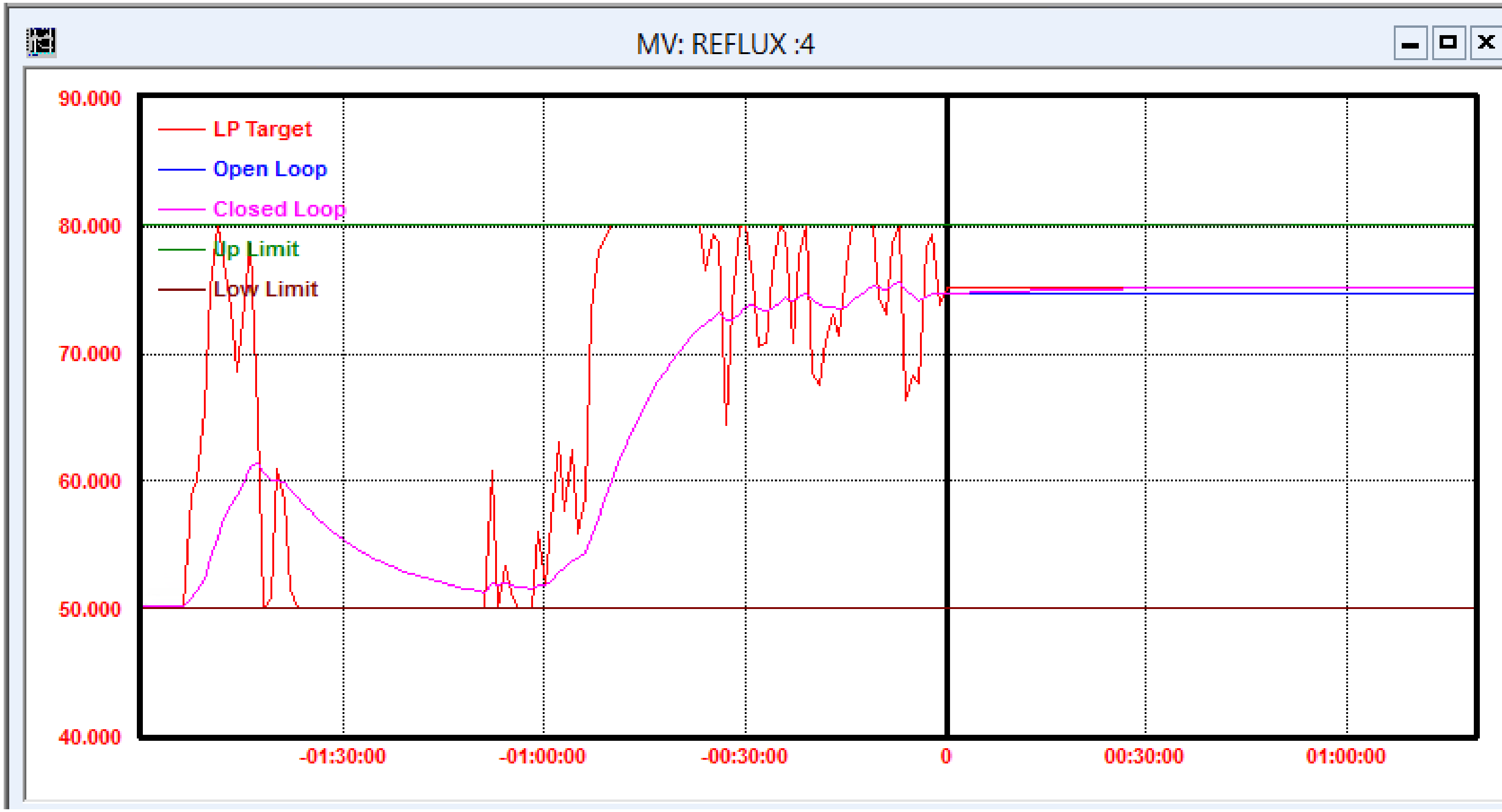
Keep Discovering

What we don't like….



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Outline

1. What is collinearity ?
2. What makes a system collinear ?
3. How does a near collinear controller behave ?
4. How does a perfect collinear controller behave ?
5. How does an uncollinear controller behave ?
6. When should we expect collinearities in :
    - Heat exchange
    - Refrigeration
    - Distillation

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What is Collinearity ?

	CV1	CV2
MV1	a	b
MV2	c	d

G = 2x2 matrix of steady state gains

G is ”collinear” when the terms are in perfect ratio : $a/c = b/d$

Collinear also known as *linearly dependent, parallel, singular*

For collinear G, there is no SS solution for ΔMVs to satisfy all ΔCVs

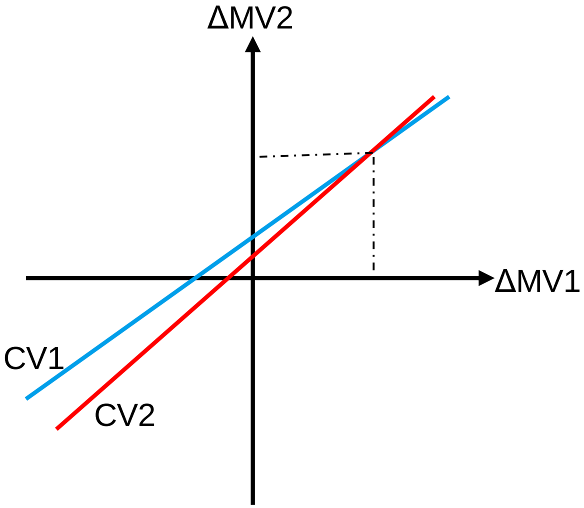
What is a Near Collinearity ?

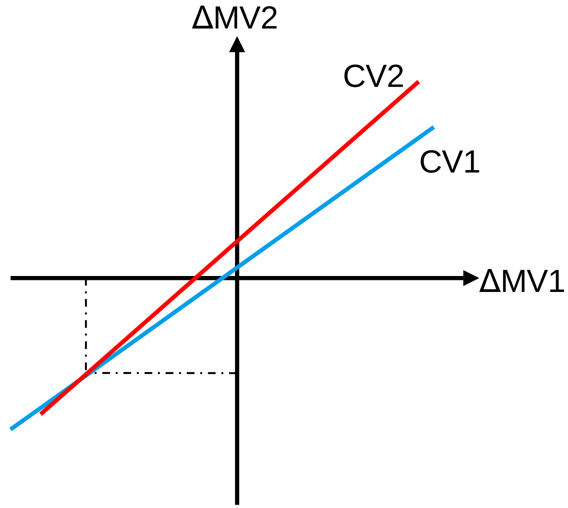
	CV1	CV2
MV1	a	b
MV2	c	d

G is ”nearly collinear” when the ratios are almost equal : a/c ≈ b/d

Near Collinear also known as *ill conditioned*

For near collinear *G*, the SS solution for ΔMVs is sensitive to noise in CVs





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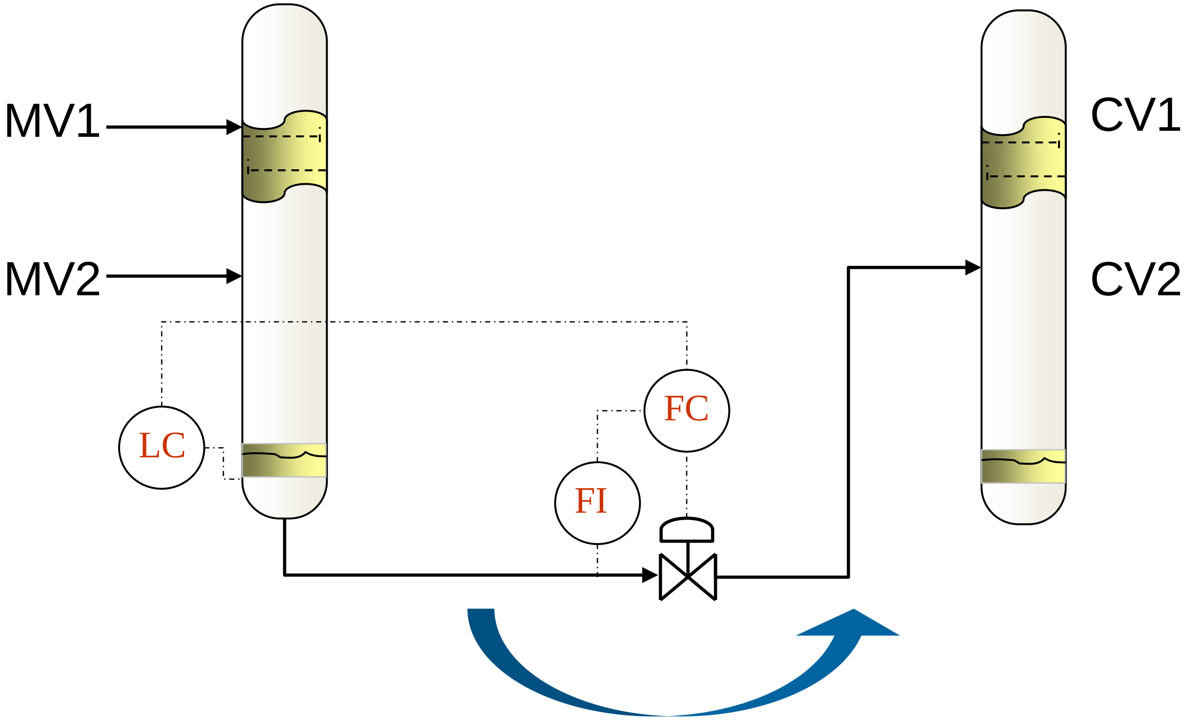
What makes a System Collinear ?



Convolution through FI

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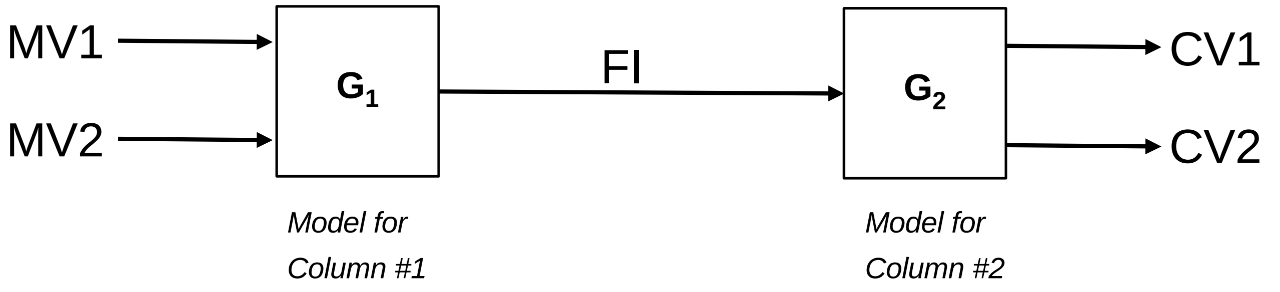
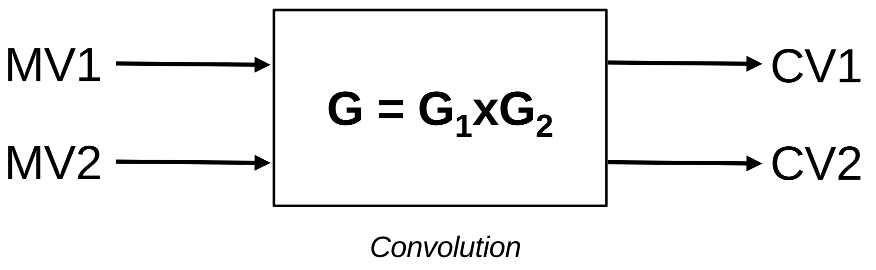
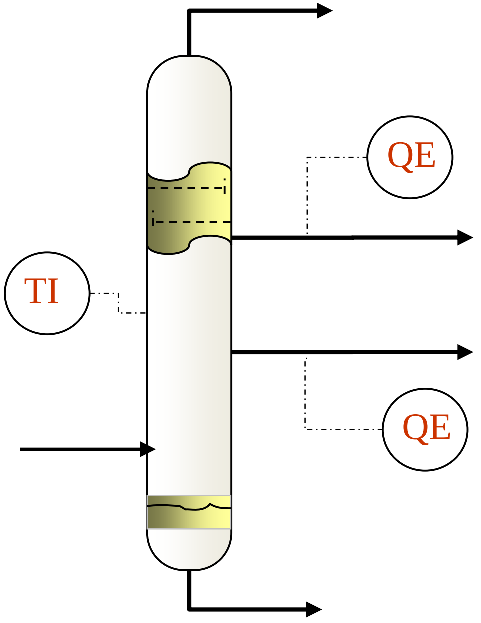
Collinearity from Convolution





*Overall Process model **G** is COLLINEAR !*

Collinearity from State Space



State space model, no direct ***D*** term :

$$\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$$

*If **C** is linear dependent , the overall process model **G** is COLLINEAR !*

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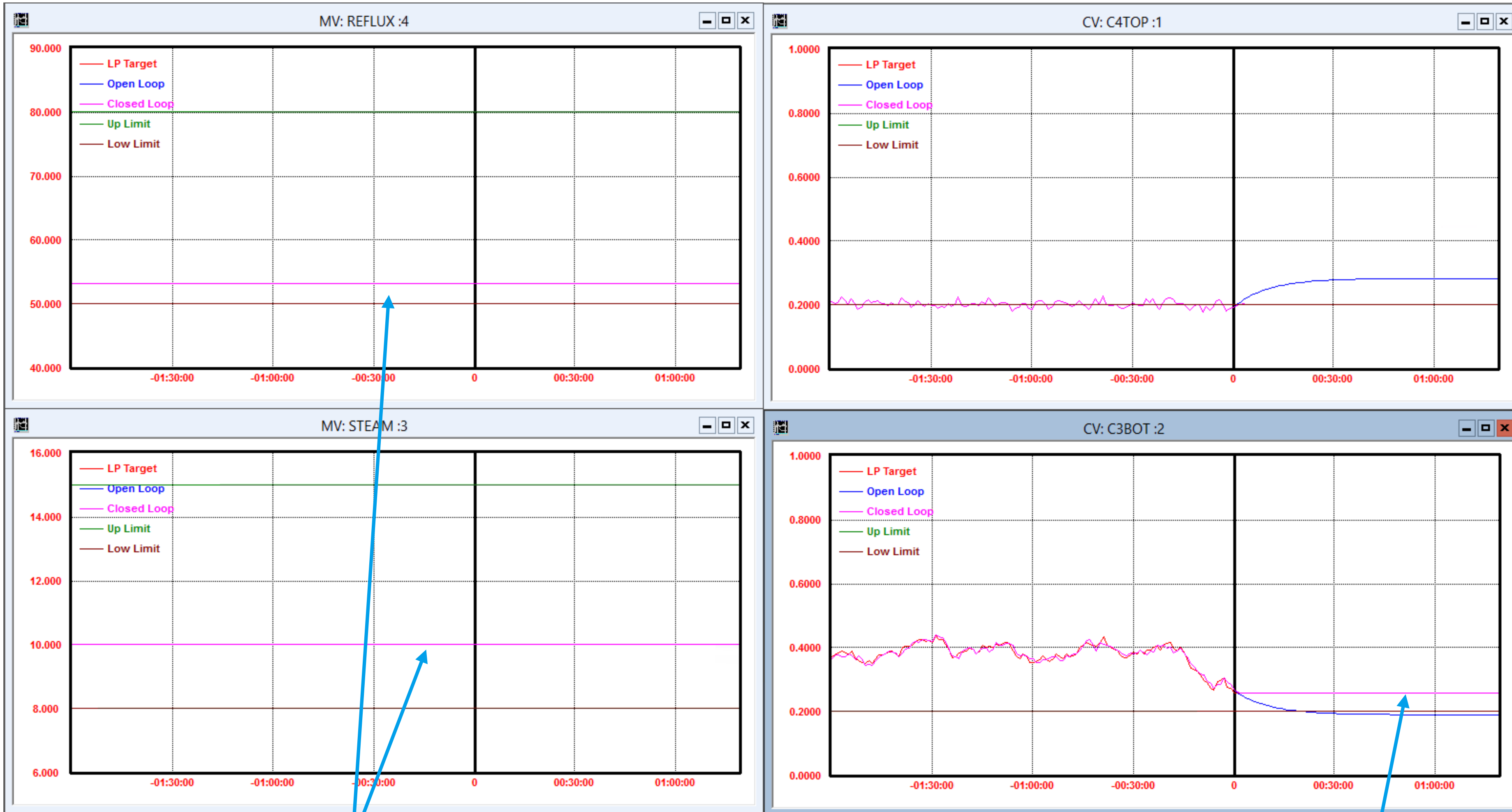
Collinear Controller, Collinear Process



Stable MVs

Give up ONE CV

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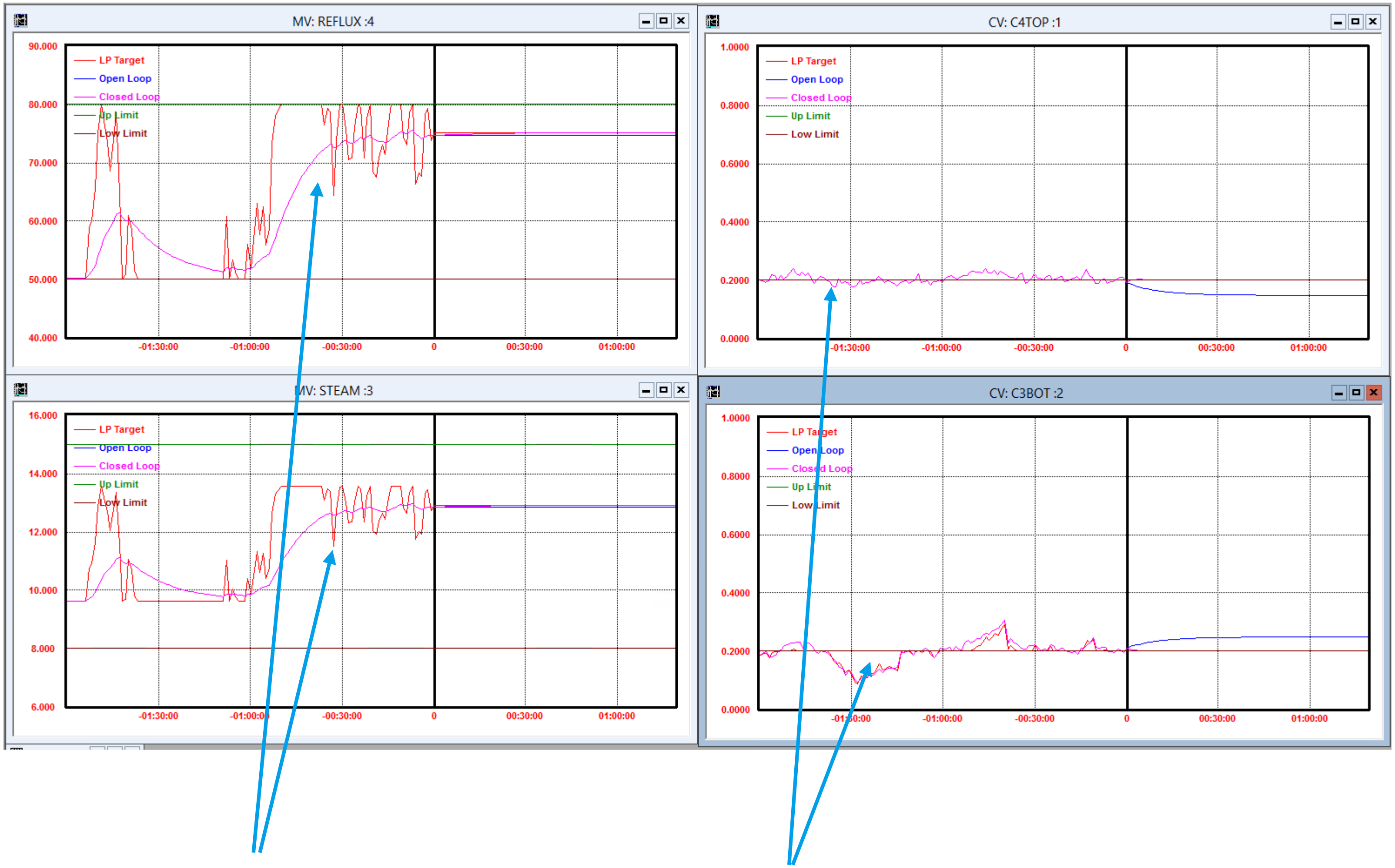
Near Collinear Controller, Collinear Process



Erratic SS solution !

Same Noise on CVs as before

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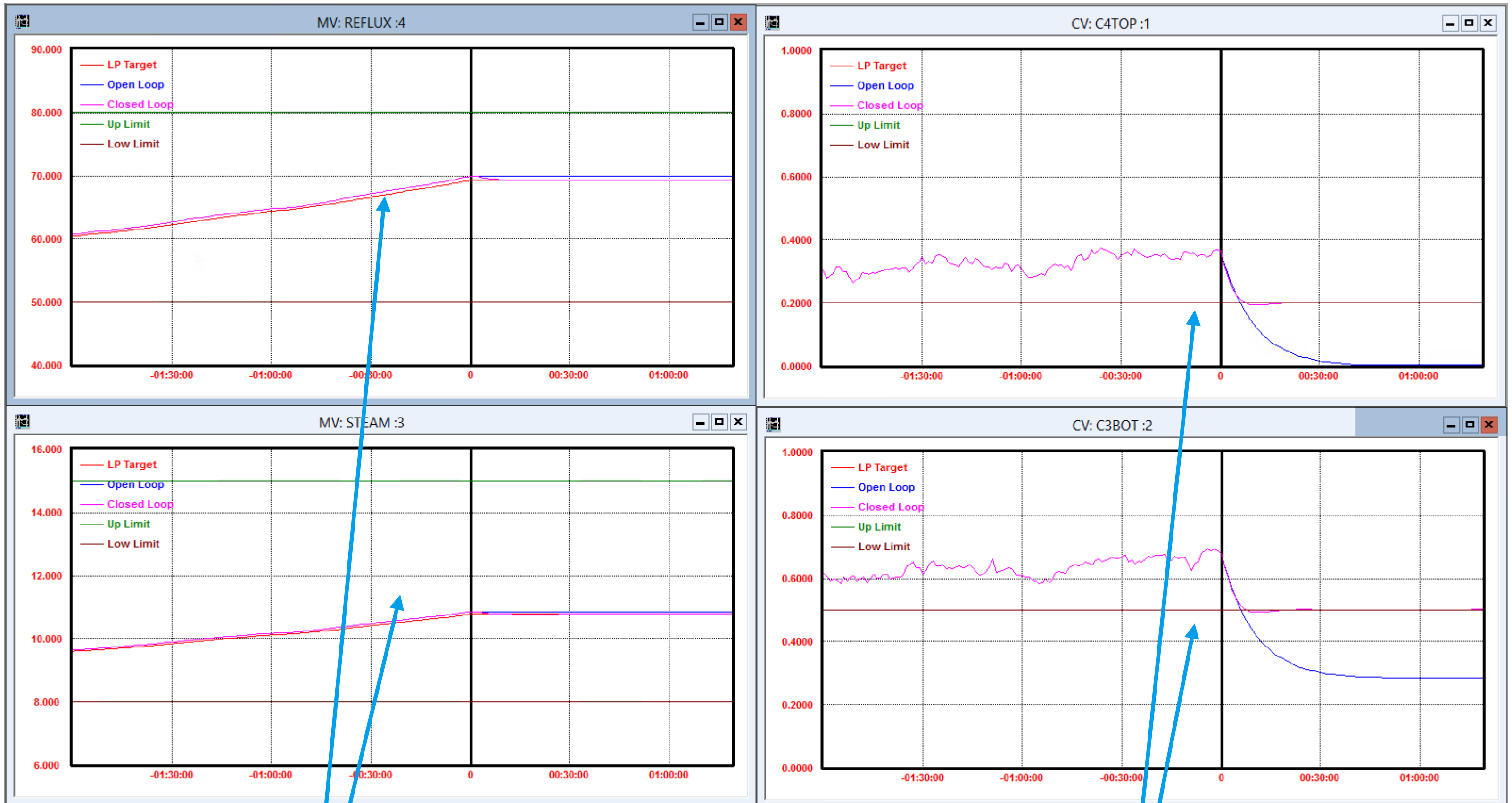
Uncollinear Controller, Collinear Process



MV ramping

CV targets never reached

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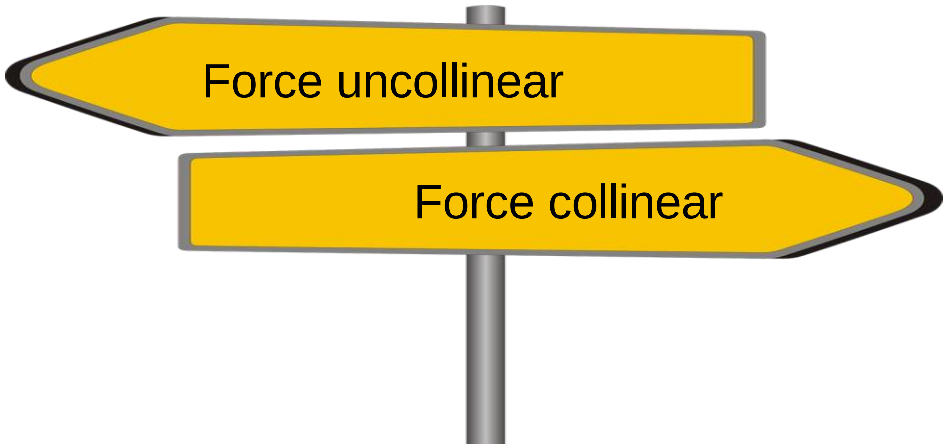
Collinearity Metrics : Condition & RGA Numbers

Comfortably Uncollinear
Cond # < 20
RGA # < 5

Near Collinear

Perfectly Collinear
Cond # = ∞
RGA # = ∞

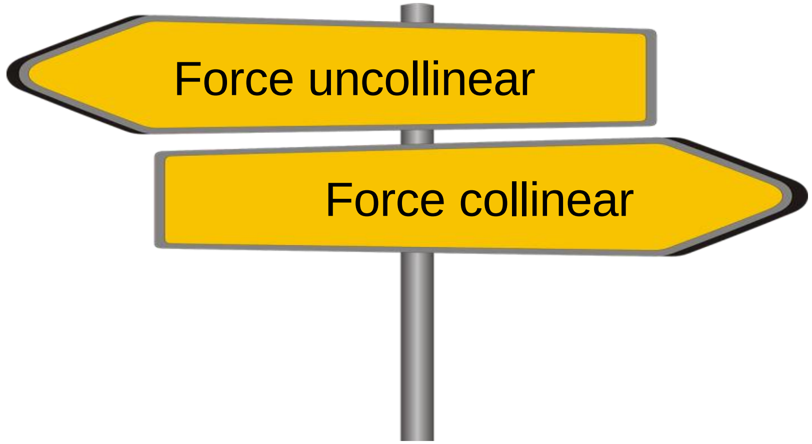


Gain Fixing

	CV1	CV2
MV1	-2.45	1.69
MV2	17.18	-14.57

RGA # = 5, Cond # = 19

UNCOLLINEAR



	CV1	CV2
MV1	-2.23	1.88
MV2	16.948	-14.288

RGA # = ∞, Cond # = ∞

PERFECT COLLINEAR

	CV1	CV2
MV1	-2.23	1.88
MV2	16.92	-14.3

Raw identified matrix

RGA # = 400, Cond # = 1600

NEAR COLLINEAR

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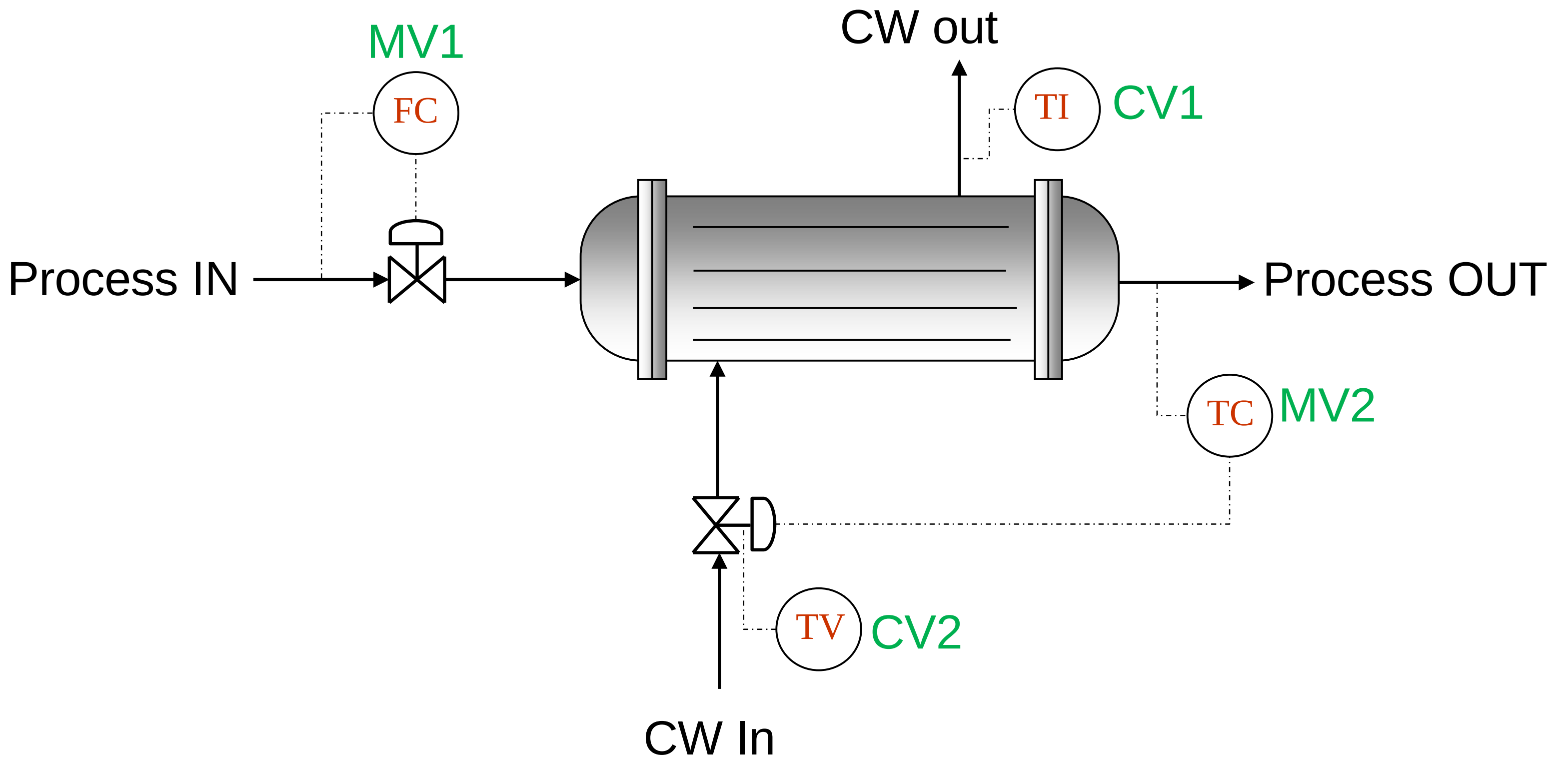
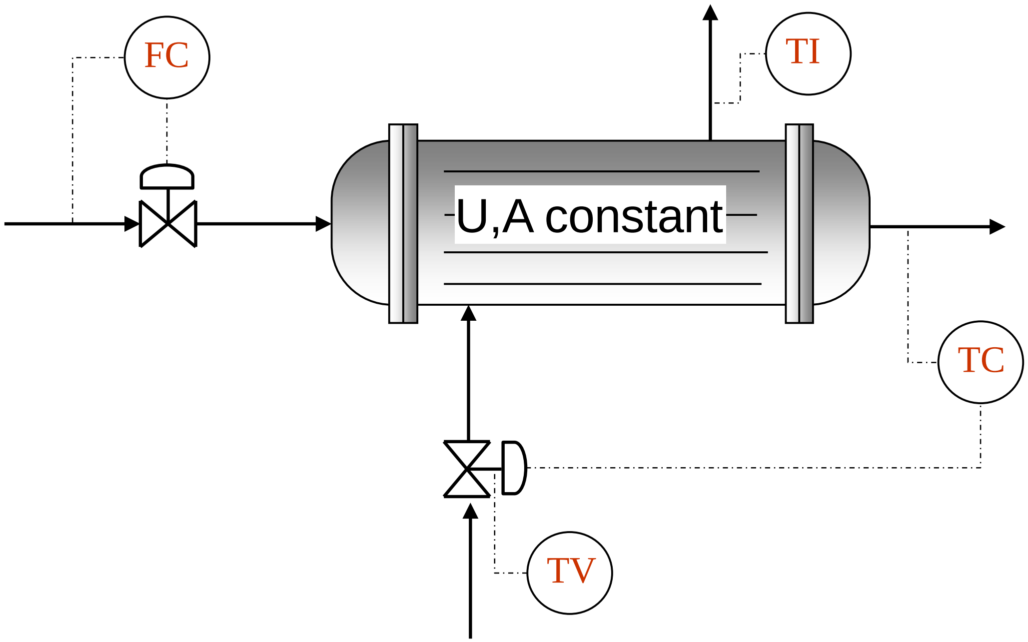
Example 1 : Heat Exchanger



Heat Exchanger, Constant U



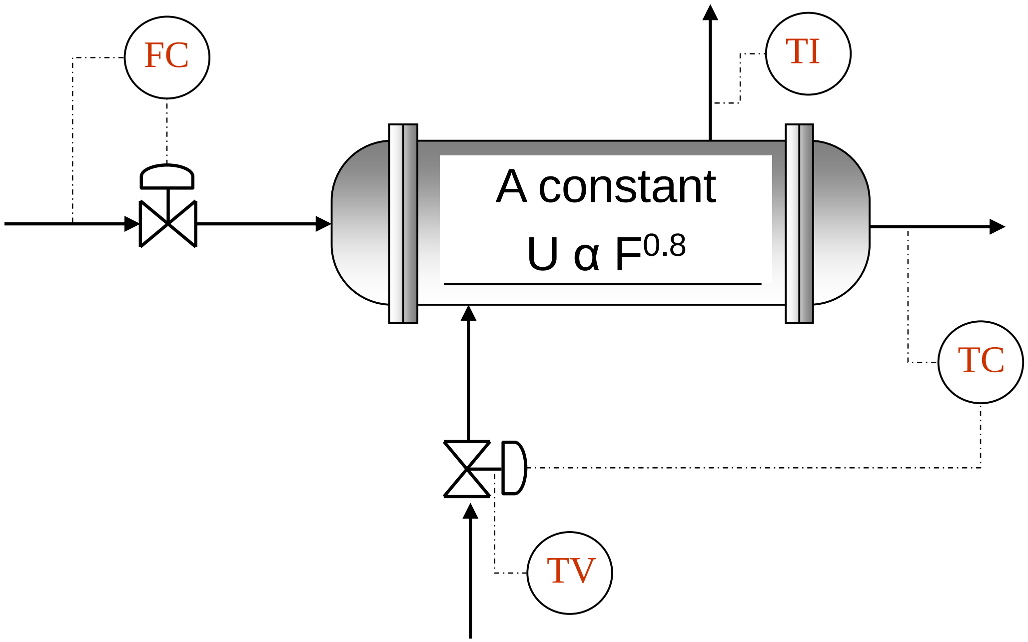
GAIN MATRIX	CW temp (C)	CW valve (%)
Process flow (t/h)	-3.75	7.35
Process temp (C)	9.00	-17.64

Determinant = -3.75×-17.64 – 7.35×9.00 = 0

System is COLLINEAR

When fouled, must give up on ONE CV (e.g. CW temp)

Heat Exchanger, Variable U



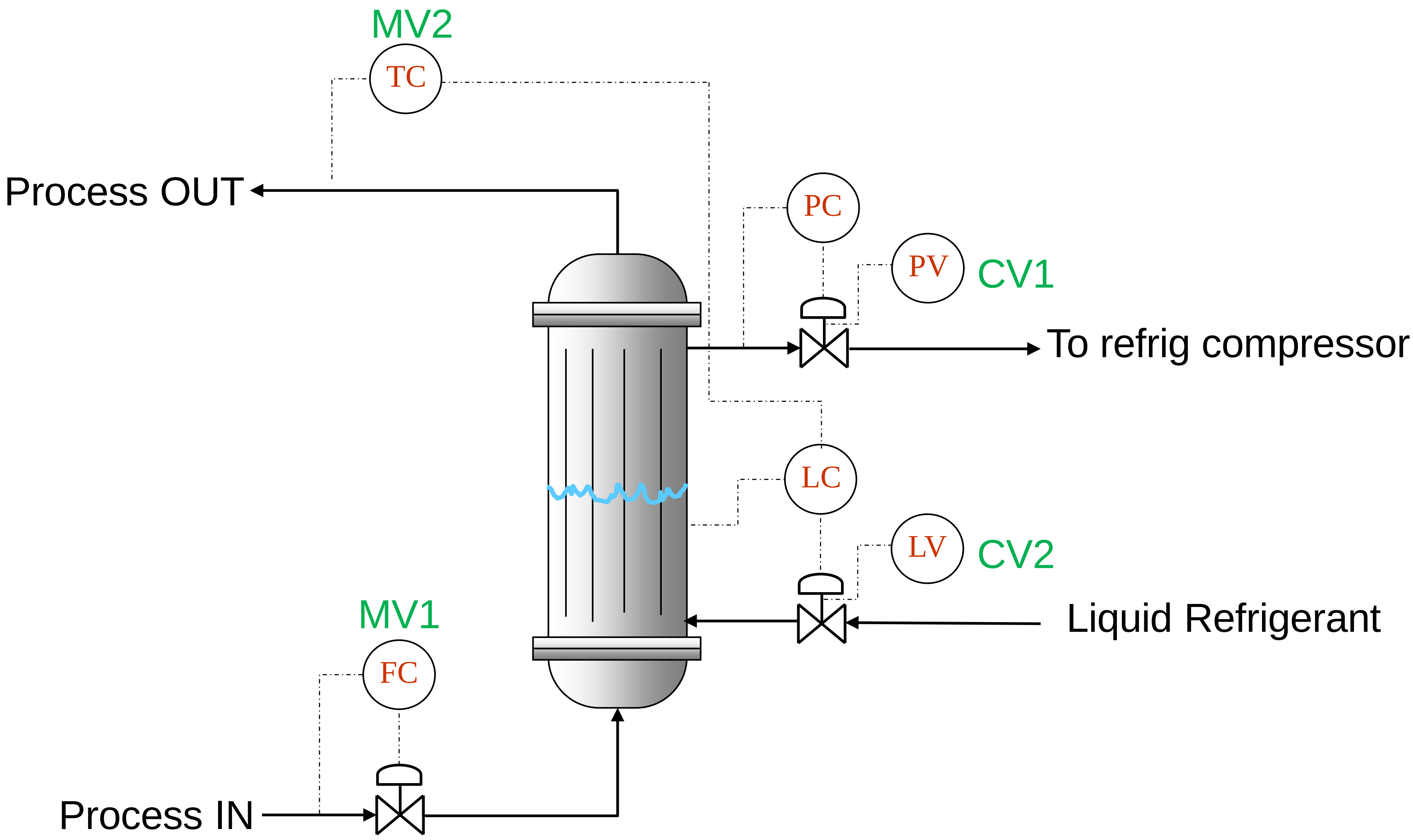
GAIN MATRIX	CW temp (C)	CW valve (%)
Process flow (t/h)	-0.1	0.99
Process temp (C)	0.35	-2.37

RGA # = 2.2, Condition # = 10.0

System is NOT COLLINEAR

For clean exchanger, both CVs can be met simultaneously

Example 2 : Refrigeration



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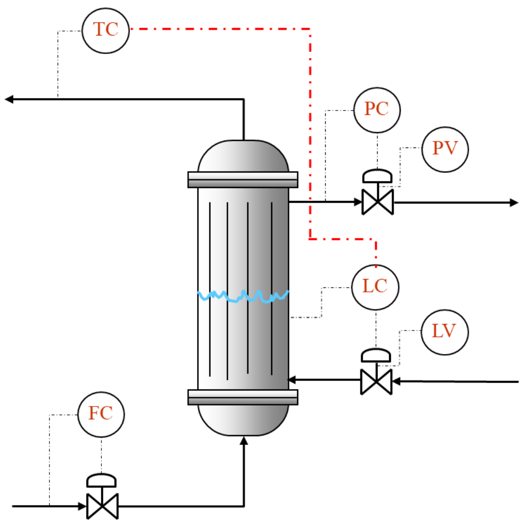
Refrigeration TC/LC Cascade



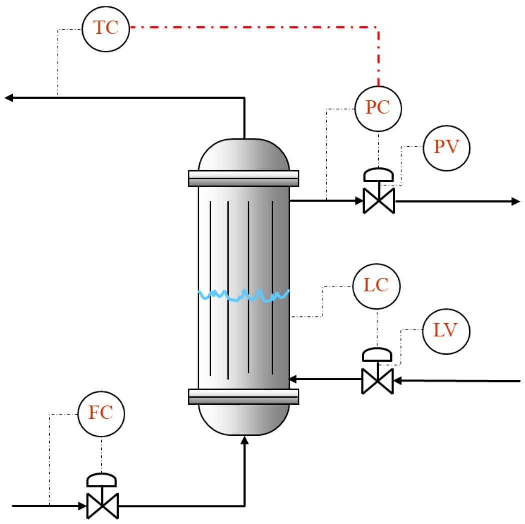
GAIN MATRIX	Refrig valve (%)	Return valve (%)
Process flow (t/h)	9.1	16.12
Process temp (C)	-1.75	-3.1

Determinant = 9.1×-3.1 – 16.12×-1.75 = 0

System is COLLINEAR

Duty increase prohibited after ONE valve hits constraint

Refrigeration TC/PC Cascade



GAIN MATRIX	Refrig valve (%)	Return valve (%)
Process flow (t/h)	-21.9	19.9
Process temp (C)	11.8	-4.7

RGA # = 0.8, Condition # = 4.9

System is NOT COLLINEAR

Duty can be maximised by operating both valves at constraint

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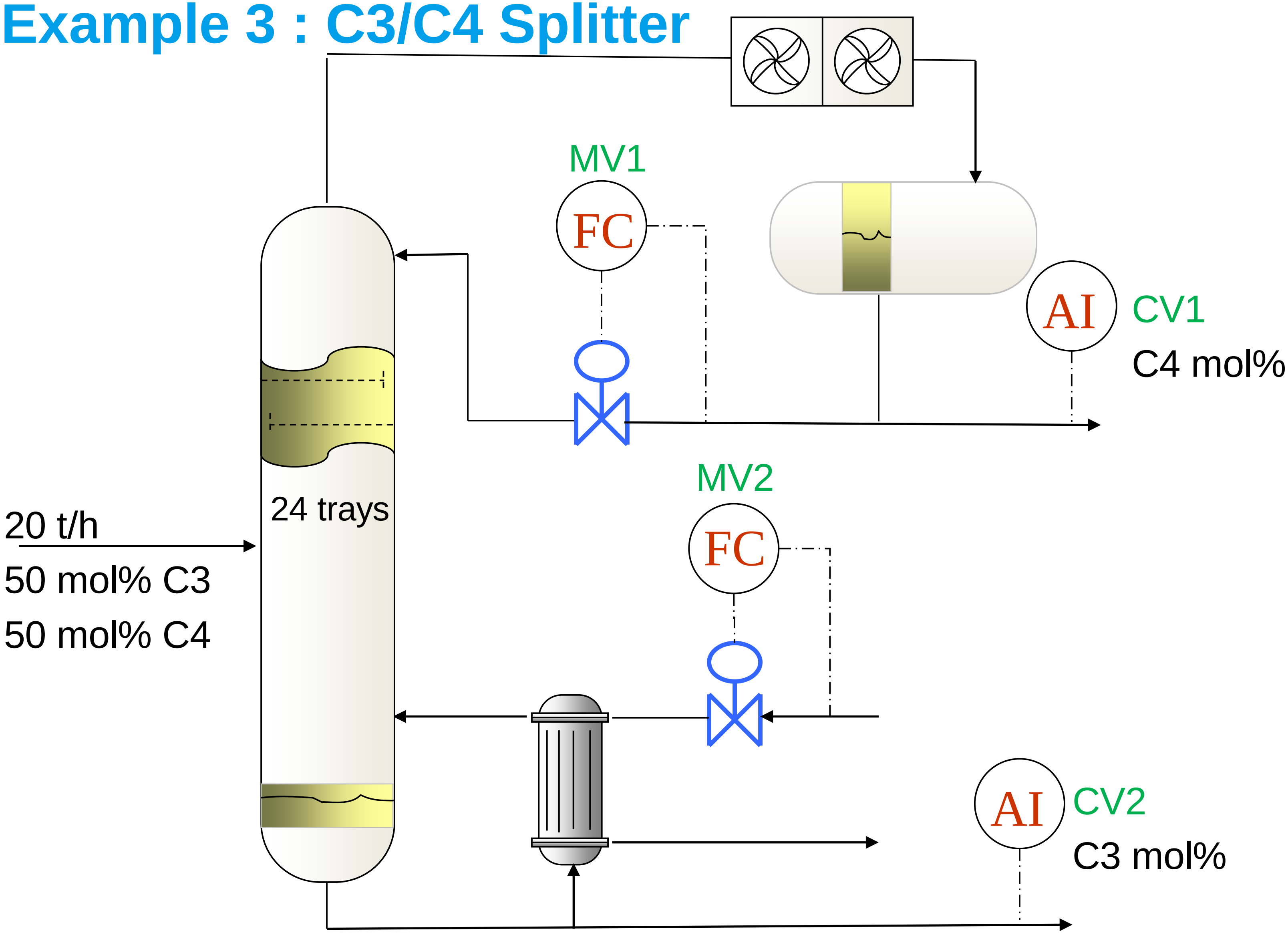
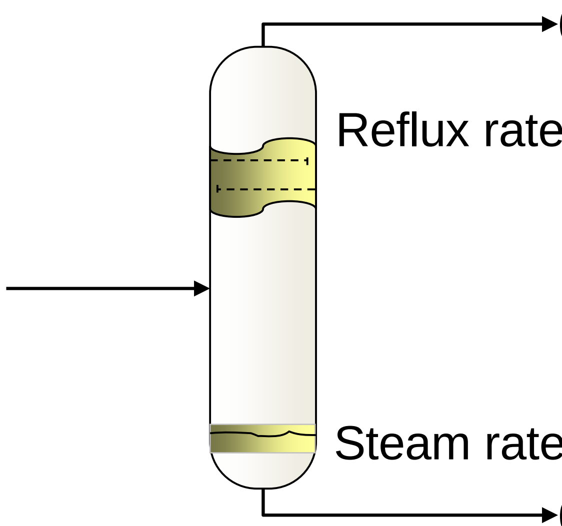
Example 3 : C3/C4 Splitter



Splitter with High RR



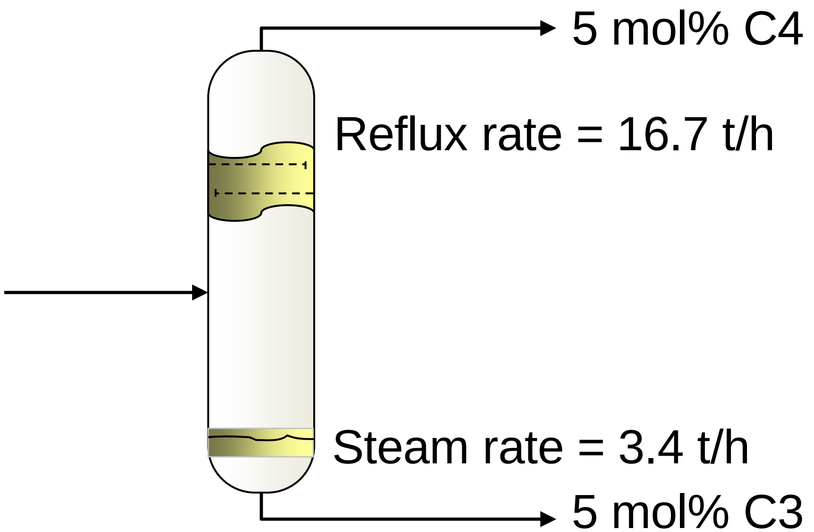
GAIN MATRIX	C4 in OH (mol%)	C3 in bottom (mol%)
Reflux rate (t/h)	-2.23	1.88
Steam rate (t/h)	16.92	-14.288

Determinant = -2.23×-14.288 – 1.88×16.92 = 0

System is COLLINEAR

Only one quality spec can met

Splitter with Low RR



GAIN MATRIX	C4 in OH (mol %)	C3 in bottom (mol %)
Reflux rate (t/h)	-3.5	3.4
Steam rate (t/h)	18.4	-33.8

RGA # = 2.1, Condition # = 6.2

System is NOT COLLINEAR

Specs on both top and bottom quality can be met simultaneously

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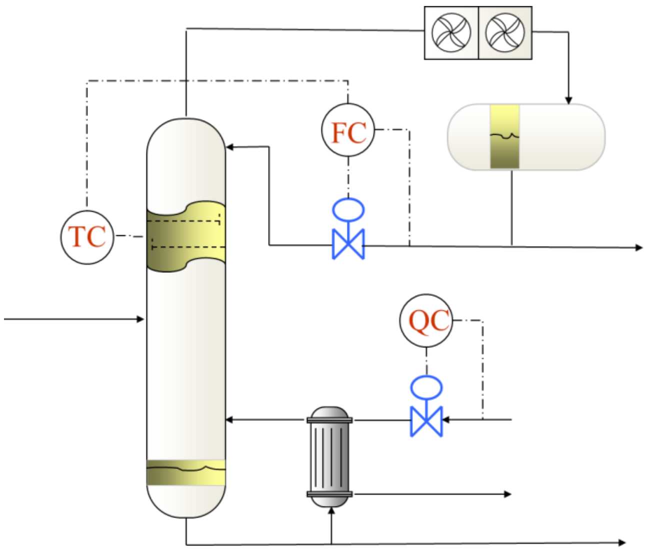
Splitter with TC



GAIN MATRIX	C4 in OH (mol%)	C3 in bottom (mol%)
Temperature (C)	+a	-b
Reboil duty (kW)	-c	-d

Determinant = (+a×-d) – (-b×-c) ≠ 0

System is NEVER COLLINEAR

Temperature + fractionation MVs circumvent the collinearity dilemma !

Conclusions

1. A collinear system is one with equal gain ratios
2. MIMO models generated by convolution are collinear
3. Condition # and RGA # measure proximity to collinearity
4. A near collinear controller gives erratic closed loop behaviour
5. A collinear controller on an uncollinear process will give up on a constraint
6. An uncollinear controller on a collinear process will ramp MVs to saturation
7. The collinearity property of a process can change depending on :
   - Process conditions e.g. fouling in heat exchangers
   - Operating point e.g. Reflux ratio in distillation
   - Base layer controls e.g. TC slave in refrigeration
8. Temperature + fractionation MVs circumvent the collinearity dilemma

Thank you

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